Relative Efficiency of AD Valorem and Unit Taxes: The Case of Endogenous Quality

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RELATIVE EFFICIENCY OF AD VALOREM AND UNIT TAXES: THE CASE OF ENDOGENOUS QUALITY

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This article studies the relative efficiency of unit and ad valorem taxes in a competitive market in which the quality of product is endogenous. The author finds that the relative efficiency of these two taxes depends on exactly how individuals value the quality of a product. Specifically, the unit tax welfare dominates the ad valorem tax in the "casket" case and the "full-price" case, whereas the ad valorem tax welfare dominates the unit tax in the "lightbulb" case.

Keywords: *excise taxes; quality; efficiency*

1. INTRODUCTION

An excise tax can take two basic forms: a unit tax based on quantity sold and an ad valorem tax based on the value of sales. These two versions of excise taxes are equivalent in a perfectly competitive market when the quality of the product in the market is exogenous.[1] Barzel (1976) introduced a quality dimension in studying the effects of an excise tax and theorized that the unit version of an excise tax should increase the average product quality in a market, whereas the ad valorem version may leave the quality unchanged. Although other studies—Bohanon and Van Cott (1991) and Kay and Keen (1991) in particular—pointed to the fact that the exact quality effect of a unit tax or an ad valorem tax also depends on the substitutability of quantity and quality in generating consumer utility, they continued to support the basic

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proposition that, other things equal, equilibrium quality in a market is higher under the unit tax than under the ad valorem tax. Clearly, it is not appropriate to hold quality fixed when comparing the two taxes' implications for welfare.

This article studies the relative efficiency of unit and ad valorem taxes in a competitive market in which the quality of product is endogenous. It finds that the relative efficiency of these two taxes depends on exactly how individuals value the quality of a product. Specifically, it compares the two taxes under three alternative quality specifications: the "casket" specification, the "full-price" specification, and the "lightbulb" specification.[2] It finds that the unit tax welfare dominates the ad valorem tax in the casket case and the full-price case, but the ad valorem tax welfare dominates the unit tax in the lightbulb case. Although these three specifications of product quality are special because they each correspond to a distinct degree of substitutability between quality and quantity in a continuum, many aspects of quality—durability, reliability, waiting time, and certain product bundling problems—can be put into these categories (Saving 1982).

2. A FRAMEWORK FOR COMPARING ALTERNATIVE EXCISE TAXES: THE CASE OF ENDOGENOUS QUALITY

To focus on the comparative welfare effects of alternative excise taxes through their impact on quality, I conduct my analysis for a perfectly competitive market with homogenous individuals. When both the quantity and the quality of a consumption good determine consumers' satisfaction, a representative individual's utility maximization problem is to maximize $U(x, q, y)$ subject to $px + y = m$, where x and q are, respectively, the quantity and quality of a taxed consumption good (referred to as x-good), y is consumption of a composite numeraire good that is not subject to any tax, p is the price of x-good, and m is income. The associated demand and indirect utility functions are denoted as $x(p, q)$ and $V(p, q)$, respectively.[3] Assuming a constant cost industry for the x-good's production, perfect competition implies that the long-run industry marginal cost is constant for any chosen q, which can be denoted as $L(q)$. It further assumes $L'(q) \geq 0$ and $L''(q) \geq 0$.

With a unit tax t_u and an ad valorem tax t_{av} simultaneously imposed on this market, the market equilibrium price and quality, $p(t_u, t_{av})$ and $q(t_u, t_{av})$, are determined by the following two conditions. The first one is a zero-profit condition resulting from competition among profit maximizing firms, that is,

$$L(q) = (1 - t_{av})p - t_u. \tag{1}$$

Another condition, also due to firms' competition, is that the equilibrium price and quality will maximize $V(p, q)$ subject to Equation 1, yielding the first-order condition

$$V_p L'(q) + V_q(1 - t_{av}) = 0. \tag{2}$$

Tax revenue collected from each individual is

$$R(t_u, t_{av}) = x(p(t_u, t_{av}), q(t_u, t_{av}))[p(t_u, t_{av})t_{av} + t_u]. \tag{3}$$

Consider the welfare effect of a change in the tax structure in the direction of using more of the unit component while the tax revenue is kept the same (with a corresponding adjustment in the ad valorem component). The following Lemma will play a critical role in proving the main results of the article.

Lemma 1. A revenue-neutral increase in the unit tax (accompanied by a corresponding decrease in the ad valorem tax) enhances welfare if and only if

$$\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u} < 0. \tag{4}$$

Proof: The welfare impact of a revenue-neutral increase in the unit tax is measured by

$$\frac{dV}{dt_u} = V_p\left(\frac{\partial p}{\partial t_u} + \frac{\partial P}{\partial t_{av}}\frac{dt_{av}}{dt_u}\right) + V_q\left(\frac{\partial q}{\partial t_u} + \frac{\partial q}{\partial t_{av}}\frac{dt_{av}}{dt_u}\right), \tag{5}$$

where

$$\frac{dt_{av}}{dt_u} = -\frac{\dfrac{\partial R}{\partial t_u}}{\dfrac{\partial R}{\partial t_{av}}}.$$

Getting comparative statics from equilibrium conditions in Equations1 and 2 and substituting them into Equation 5, one has

$$\frac{dV}{dt_u} = \frac{V_p}{(1-t_{av})\partial R / \partial t_{av}}\left(\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u}\right). \tag{6}$$

Because $V_p < 0$, and $\frac{\partial R}{\partial t_{av}} > 0$,[4] Equation 6 implies that a revenue-neutral increase in the unit tax enhances welfare if and only if Equation 4 is satisfied. Q.E.D.

The intuition for Lemma 1 is as follows. Given tax parameters t_u and t_{av}, perfect competition implies a realized consumer welfare

$$W(t_u, t_{av}) \equiv \max_{p,q} V(p,q)$$
$$s.t\ L(q) = (1-t_{av})p - t_u.$$

Applying the envelope theorem, one has

$$\frac{\partial W}{\partial t_{av}} = p\frac{\partial W}{\partial t_u},$$

which, together with Equation 4, implies that a compensated increase in the unit component of the tax structure (accompanied by a reduction in the ad valorem component so that the consumer welfare remains the same) would generate a higher tax revenue. Thus, the unit tax is more efficient at the margin when Equation 4 is satisfied.

The determinant $\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u}$ can be further expressed as

$$\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u} = (pt_{av} + t_u)\left[\frac{\partial x}{\partial p}\left(\frac{\partial p}{\partial t_{av}} - p\frac{\partial p}{\partial t_u}\right) + \frac{\partial x}{\partial q}\left(\frac{\partial q}{\partial t_{av}} - p\frac{\partial q}{\partial t_u}\right)\right]$$
$$+ x(p,q)t_{av}\left(\frac{\partial p}{\partial t_{av}} - p\frac{\partial p}{\partial t_u}\right). \tag{7}$$

From Equation 7, two observations are in order with respect to the welfare comparison between ad valorem and unit taxes. First, $\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u} = 0$, that is, the two taxes are equally efficient, if the quality of the taxed product is exogenous. To see this, just note that q does not change with either tax rate and that $\frac{\partial p}{\partial t_{av}} - p\frac{\partial p}{\partial t_u} = 0$ from Equation 1 where $L(q)$ is now a constant. Second, the relative efficiency of the two excise taxes depends on how consumers value quantity and quality characteristics of the taxed product at the margin, which determines the properties of demand and comparative statics results. In addition, which tax has an efficiency edge at the margin may also depend on the values of t_{av} and t_u. That is, the two excise taxes may not dominate each other on the whole, and there might exist an optimal mix of both taxes in collecting a certain tax revenue.

Nonetheless, we will show in the following that when quality specification assumes one of the three often-discussed forms, clear-cut welfare dominance can be obtained. Importantly, either tax can dominate, depending on exactly how consumers value the quality aspect of a product.

3. RELATIVE EFFICIENCY OF EXCISE TAXES UNDER THREE QUALITY SPECIFICATIONS

UNIT TAX DOMINATES: THE FULL-PRICE SPECIFICATION

A quality specification known as the full-price case captures a category of situations in which an improvement in quality is exactly equivalent to a reduction in price such that the indirect utility function has the form

$$V(p, q) \equiv F(p - H(q)), \tag{8}$$

where $H'(q) > 0$ and $H''(q) < 0$, capturing diminishing marginal valuation of quality. From Roy's Identity, the demand for the x-good has the form

$$x(p, q) \equiv G(p - H(q)). \tag{9}$$

This quality specification is relevant when the main quality concern with a product or service is the waiting time associated with its acquisition/consumption or when the quality of a product is increased solely by bundling with it some other products.[5] Such waiting time or added products can be assumed to have a price established in a competitive labor or product market, and consumers would be willing to pay a higher price for the same product or bundle of products with a shorter waiting time or more attachments. Thus, for the case of waiting time, $H(q)$ is the market monetary value of saved time associated with consuming each unit of the x-good and $p - H(q)$ can be interpreted as the full price.

For the full-price specification, Equation 2 becomes

$$L'(q) - (1 - t_{av})H'(q) = 0, \tag{2'}$$

which, in combination with Equation 1, yields the following comparative statics relations:

$$\begin{aligned}
\frac{\partial q}{\partial t_u} &= 0 \\
\frac{\partial q}{\partial t_{av}} &= \frac{-H'(q)}{L''(q) - (1 - t_{av})H''(q)} < 0 \\
\frac{\partial p}{\partial t_u} &= \frac{1}{1 - t_{av}} > 0 \\
\frac{\partial p}{\partial t_{av}} &= \frac{L'(q)\frac{\partial q}{\partial t_{av}} + p}{1 - t_{av}}
\end{aligned} \tag{10}$$

Using these comparative statics results, we have from Equation 7 that

$$\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u} = \frac{t_{av}GL'(q)\frac{\partial q}{\partial t_{av}}}{1 - t_{av}} < 0.$$

Thus, according to Lemma 1, in the full-price case, substituting the unit tax for the ad valorem tax at the margin increases consumers' welfare as long as t_{av} is positive. So a welfare-maximizing government should use the unit tax alone in a market of this type. A simple reason for the welfare dominance of the unit taxation in this case is that pro-

ducers cannot avoid paying unit taxes by altering product quality (the waiting time, for example), but they can reduce their tax liabilities under the ad valorem taxation by making consumers wait longer and pay a lower price. Thus, in a sense, the ad valorem taxation distorts the firms' quality choice, whereas the unit taxation does not.[6]

UNIT TAX DOMINATES: THE CASKET SPECIFICATION

Another quality specification discussed in Bohanon and Van Cott (1991) and Kay and Keen (1991), known as the casket case, covers a range of products that are characterized by a one-or-none consumption pattern. A perfect example in this category is the funeral casket. As observed by Bohanon and Van Cott, people buy one casket or none for the deceased in their family, and there is no quantity-quality substitutability in that they "do not purchase a partial but more durable casket, nor are they willing to trade casket durability for more than one casket" (p. 234). Other examples in this casket category include housing units and car insurance coverage.

Given that price is not too high and quality not too low for a casket-type product, the consumer's one-or-none decision will generate a one-unit fixed demand. That is, the demand for the product is fixed at one unit regardless of the price and the quality of the product as long as they fall in a reasonable range. Formally,

$$x(p, q) \equiv 1. \tag{11}$$

Using Roy's Identity to recover the indirect utility affords the following general form:

$$V(p, q, m) \equiv F(p - m - H(q)). \tag{12}$$

Note the similarity between Equations 12 and 8 and that Equation 11 is a special case of Equation 9 in which $G(\bullet) \equiv 1$.[7] Obviously, abstract from the "to buy or not to buy" decision, the casket specification can be regarded as a special case of the full-price specification in which the quantity demanded is invariant with price and quality.

Of course, then, the results derived under the full-price specification also apply to the casket specification. In particular, the equilib-

rium quality will not change with an increase in the unit tax but will decline with an increase in the ad valorem tax, and the unit tax welfare dominates the ad valorem tax.

AD VALOREM TAX DOMINATES: THE LIGHTBULB SPECIFICATION

The lightbulb quality specification covers a wide rage of situations in which utility can be written as a function of quality-adjusted consumption, that is, $U(xH(q), y)$. As $H(q)$ measures the services provided by each unit of the commodity, the effective price for one unit of services is $p/H(q)$. Hence, the indirect utility function for the lightbulb specification has the form

$$V(p, q) \equiv F(p / H(q)), \tag{13}$$

where, again, $H'(q) > 0$ and $H''(q) < 0$. Therefore, the demand for the x-good has the form

$$x(p, q) \equiv (1 / H(q))G(p / H(q)). \tag{14}$$

This quality specification is relevant when quality enters the picture as durability or reliability. A perfect example of a product with durability as the main concern is the lightbulb. Consumers would think one lightbulb with a use life of 2,000 hours as being as good as two lightbulbs that each have a use life of 1,000 hours.[8] To show that reliability can also be characterized by the lightbulb case, I borrow an argument forwarded by Saving (1982). Suppose that a certain product provides a given level of service with probability q and fails with $1 - q$ in which no service is provided. Assume that consumers are risk neutral with respect to the services provided by this product. Then the expected utility from x units of this product is exactly the utility of expected consumption, xq, yielding a lightbulb-type direct utility $U(xq, y)$.

In the lightbulb case, Equation 2 becomes

$$H(q)L'(q) - (1 - t_{av})pH'(q) = 0, \tag{2''}$$

which, in combination with Equation 1, yields the following comparative statics relations:

$$\begin{aligned}
\frac{\partial q}{\partial t_u} &= \frac{1}{H(q)L''(q) - [L(q) - t_u]H''(q)} > 0 \\
\frac{\partial q}{\partial t_{av}} &= 0 \\
\frac{\partial p}{\partial t_u} &= \frac{L'(q)\frac{\partial q}{\partial t_u} + 1}{1 - t_{av}} > 0 \\
\frac{\partial p}{\partial t_{av}} &= \frac{p}{1 - t_{av}} > 0.
\end{aligned} \tag{15}$$

Using these comparative statics results, one has from Equation 7 that

$$\frac{\partial R}{\partial t_{av}} - p\frac{\partial R}{\partial t_u} = \frac{t_u pGH'(q)\frac{\partial q}{\partial t_u}}{H^2(q)} > 0.$$

Thus, in the lightbulb case, substituting the unit tax for the ad valorem tax at the margin always reduces the individual's welfare as long as t_u is positive. So, a welfare-maximizing government should use the ad valorem tax alone in a market of this type. This result is intuitive because, in this case, producers can avoid tax liabilities under the unit taxation by increasing product quality, but they cannot do so under the ad valorem taxation because a price increase will accompany the quality increase.

4. CONCLUSION

Assuming exogenous quality, the existing literature has established that the two basic versions of the excise tax are equivalent in perfect competition and the ad valorem tax has an efficiency edge in noncompetitive environments. Introducing endogenous quality, this article shows that, if a market can be approximated by perfect competition, the unit tax welfare dominates for two often-used quality specifications (the casket and the full-price specifications), and the ad valorem tax does so for another widely used quality specification (the lightbulb specification). So, how individuals value the quality of a taxed good also matters in assessing the relative efficiency of the two excise taxes.

As has been demonstrated, products featuring inelastic demand (with respect to both price and quality) belong to the casket category, products whose acquisition or consumption involves a prominent time aspect and products whose composition or sales are notably characterized by bundling can be classified into the full-price category, and products with durability or reliability as their main quality concern are captured by the lightbulb category.

In general, however, to determine what exactly constitutes "quality" for a particular good is not always an easy task. Although "higher quality" generally means "more of something," the quality characteristics tend to be multidimensional and vary considerably from one commodity to another. Take cigarettes, for example. It is not obvious which above-mentioned category, if any, is applicable to the cigarette quality. Nonetheless, differential quality effects from two excise taxes can help reveal the true quality characteristics of cigarette products. Sobel and Garrett (1997) found a significant positive effect on cigarette quality from the unit tax and no quality effect from the ad valorem tax (approximated by the sales tax). This is exactly the prediction for the lightbulb case (see Equation 15). Moreover, according to Kay and Keen (1991), the lightbulb scenario is the only case in which a change in the ad valorem tax has no quality effect. Hence, one can conclude that the cigarette quality is captured by the lightbulb specification. Thus, from the welfare analysis of this article, it is the ad valorem tax, rather than the unit tax, that should be imposed in the cigarette market.

NOTES

1. It has been found that the ad valorem tax has an efficiency edge over the unit tax in various noncompetitive environments (see Suits and Musgrave 1953; Delipalla and Keen 1992; Skeath and Trandel 1994; Liu and Saving 2001). All these studies, however, assume a fixed product quality.

2. The terminology is due to Bohanon and Van Cott (1991) and Kay and Keen (1991).

3. One can omit the income argument in these functions because it is constant throughout.

4. A welfare (or revenue) maximizing government would not raise tax rates to a point at which revenue is decreasing in the tax rates. I adopt this assumption throughout this article.

5. A special form of quality-enhancing product bundling is rebate. For example, an airline company may try to increase the attractiveness of air traveling by taping $20 bills under the seats

on its airplanes, as opposed to giving a $20 discount. I want to thank a referee for suggesting this example.

6. In a certain aspect, an "entry fee" (a fee for each purchase) is like a unit tax. In a somewhat different framework, Umbeck (1980) showed that an entry fee does not distort product quality choices.

7. Also note that Equation 8 is more general than Equation 12 because Equation 8 leaves open how income enters the indirect utility function.

8. I should point out that the critical characteristic here is not durability per se but the perfect substitutability between quantity and durability in generating utility. For example, it may well be the case that the only quality concern with a funeral casket is its durability, but the funeral casket does not belong to the "lightbulb" specification here because there is no substitutability between quantity and durability for the funeral case.

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